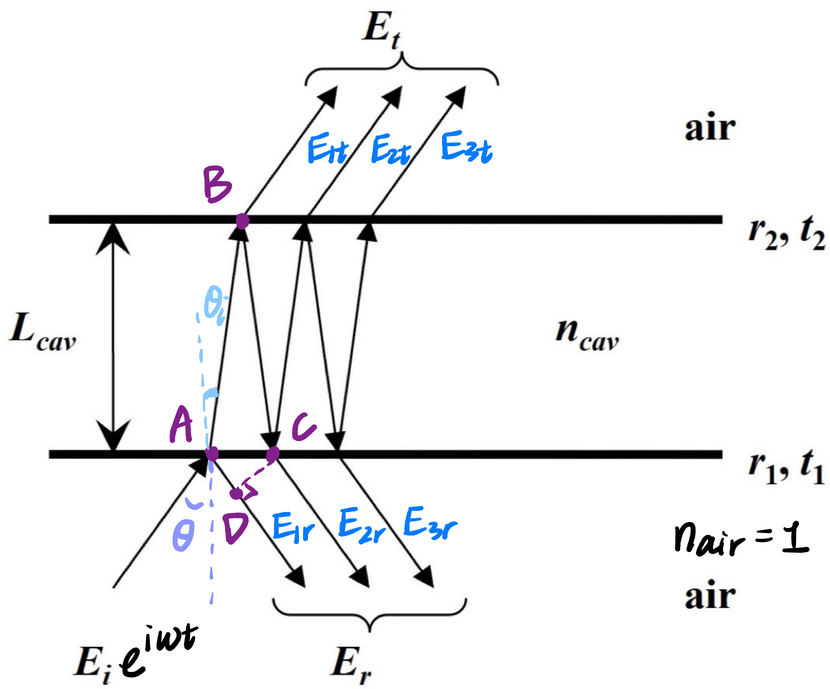
PHYS-434 Physics of photonic semiconductor devices

Series 6 - Basic notions on the physics of semiconductor microcavities

1)



We wish to determine the phase shift δ associated with the optical path difference between the first reflected beam and the second one:

$$\Lambda = n_{cav}(\overline{AB} + \overline{BC}) - \underbrace{n_{air}}_{=1} \cdot \overline{AD} \qquad \overline{AB} = \overline{BC} = \frac{L_{cav}}{\cos\theta_i}, \quad \overline{AD} = \underbrace{\overline{AC}}_{= 2L_{cav}\tan\theta_i}\sin\theta,$$

Snell law: $\sin\theta = n_{cav}\sin\theta_i$,

$$= 2n_{cav}L_{cav}\frac{1}{\cos\theta_i} - 2n_{cav}L_{cav}\frac{\sin^2\theta_i}{\cos\theta_i} = 2n_{cav}L_{cav}\cos\theta_i$$

$\Rightarrow \delta = \frac{2\pi}{\lambda}\Lambda$ (when neglecting dephasing occurring at the reflection)

$$\Rightarrow \delta = \frac{2\pi}{\lambda}(2n_{cav}L_{cav}\cos\theta_i)$$

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2) The transmitted field is related to the incoming one through:

$$E_{1t} = E_i e^{i\omega t} \cdot t_1 \cdot e^{i\delta_0} \cdot t_2 \qquad \left(\delta_0 = \frac{2\pi}{\lambda} n_{cav}\overline{AB}\right)$$

$$E_{2t} = E_i e^{i\omega t} \cdot t_1 \cdot e^{i\delta_0} \cdot (r_2 \cdot r_1 \cdot e^{i\delta}) \cdot t_2$$

$$E_{3t} = E_i e^{i\omega t} \cdot t_1 \cdot e^{i\delta_0} \cdot (r_2 \cdot r_1 \cdot e^{i\delta})^2 \cdot t_2$$

$$\vdots$$

$$E_{Nt} = E_i e^{i\omega t} \cdot t_1 \cdot e^{i\delta_0} \cdot (r_2 \cdot r_1 \cdot e^{i\delta})^{N-1} \cdot t_2$$

$$\Bigg\} \; E_t = E_{1t} + E_{2t} + E_{3t} + \dots + E_{Nt}$$

$$\Rightarrow E_t = E_{1t} \left[1 + r_1 r_2 e^{i\delta} + (r_1 r_2 e^{i\delta})^2 + \dots + (r_1 r_2 e^{i\delta})^{N-1} \right]$$

(physically $|r_1 r_2 e^{i\delta}| < 1$, provided $N \to \infty$

$$E_t = E_{1t} \left[\frac{1}{1 - r_1 r_2 e^{i\delta}} \right] \qquad E_{1t} = E_i e^{i(\omega t + \delta_0)} \cdot t_1 t_2$$

geometric series: $\sum_{k=0}^{\infty} a r^k \xrightarrow{|r|<1} \frac{a}{1-r}$

$$\Rightarrow E_t = E_i e^{i(\omega t + \delta_0)} \left[\frac{t_1 t_2}{1 - r_1 r_2 e^{i\delta}} \right]$$

3) $$T = \left| \frac{E_t}{E_i} \right|^2 = \frac{(t_1 t_2)^2}{\underbrace{|1 - r_1 r_2 \cos\delta - i r_1 r_2 \sin\delta|^2}}$$

$$(1 - r_1 r_2 \cos\delta)^2 + (r_1 r_2 \sin\delta)^2 = 1 - 2 r_1 r_2 \cos\delta + (r_1 r_2)^2$$

$$= (1 - r_1 r_2)^2 + 2 r_1 r_2 \underbrace{(1 - \cos\delta)}_{2\sin^2\frac{\delta}{2}}$$

$$\Rightarrow T = \frac{(t_1 t_2)^2}{(1 - r_1 r_2)^2 + 4 r_1 r_2 \sin^2\left(\frac{\delta}{2}\right)} \quad \text{or} \quad \frac{(t_1 t_2)^2}{1 + (r_1 r_2)^2 - 2 r_1 r_2 \cos\delta}$$

In absence of any optical losses: $R + T = 1$

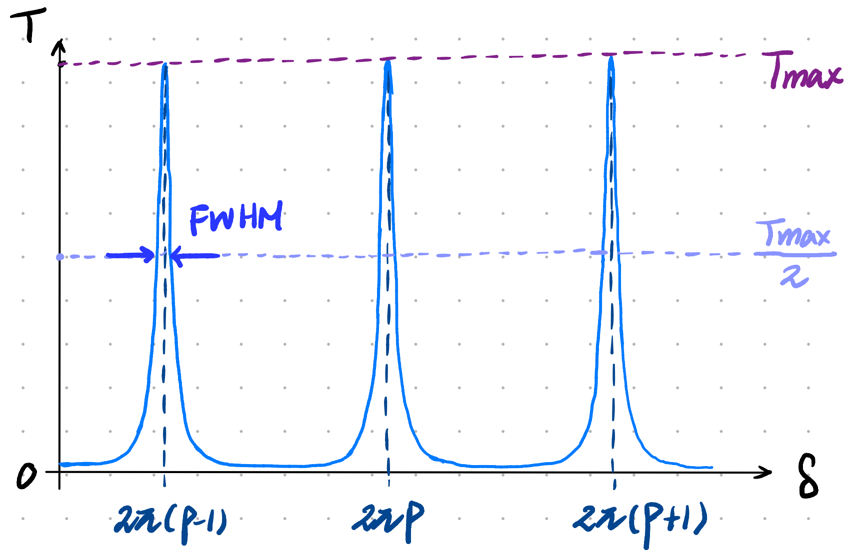
4) Condition of cavity modes: $\delta = 2\pi p$, p = integer

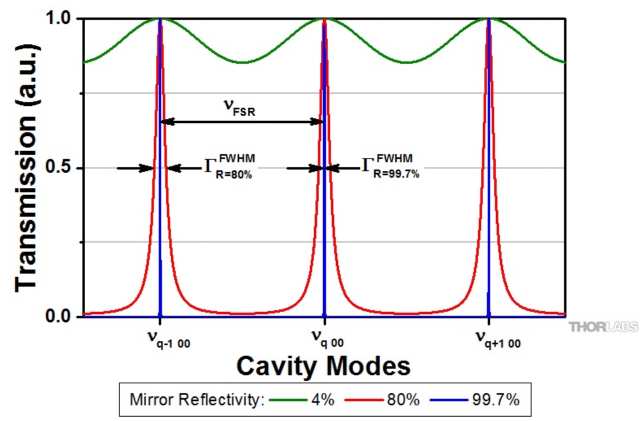
$$\Rightarrow \frac{\cancel{2\pi}}{\lambda} (2 n_{cav} L_{cav} \cos\theta_i) = \cancel{2\pi} p \Rightarrow 2 n_{cav} L_{cav} \cos\theta_i = p\lambda$$

cavity mode wavelength (associated with E)

$\oplus$ phase at the reflection (depends on energy E): $\phi(E)$

$$\Rightarrow 2n_{cav}L_{cav}\cos\theta_i = \left(p - \frac{\phi(E)}{2\pi}\right)\lambda$$

5) 



The linewidth (FWHM) of the modes γ_c is obtained by determining δ':

$T(\delta') = \frac{T_{max}}{2}$ * with $\delta' = \delta + \varepsilon$, $\delta = 2\pi p$ (p = integer), $\varepsilon \ll \delta$ *

* here, everything is calculated with $T_{min} \sim 0$, as shown in the fig. on left, which is generally the case for Fabry Perot cavities in reality ($R \sim 1$)

$$T_{max} = T(\delta) = \frac{(t_1t_2)^2}{(1-r_1r_2)^2}$$

$$\Rightarrow \frac{\cancel{(t_1t_2)^2}}{2(1-r_1r_2)^2} = \frac{\cancel{(t_1t_2)^2}}{1+(r_1r_2)^2 - 2r_1r_2\cos\delta'}$$

$$\Rightarrow \overset{1}{\cancel{2}} + \cancel{2}(r_1r_2)^2 - 4r_1r_2 = \cancel{1} + \cancel{(r_1r_2)^2} - 2r_1r_2\cos\delta'$$

$$\cos\delta' = \cos(2\pi p + \varepsilon) = \cos\varepsilon$$

$$\Rightarrow \cos\varepsilon = -\frac{(1-r_1r_2)^2 - 2r_1r_2}{2r_1r_2} = 1 - \frac{(1-r_1r_2)^2}{2r_1r_2}$$

$$\overset{\varepsilon \ll \delta}{\Rightarrow} \cancel{1} - \frac{\varepsilon^2}{\cancel{2}} = \cancel{1} - \frac{(1-r_1r_2)^2}{\cancel{2}r_1r_2} \Rightarrow \varepsilon = \frac{1-r_1r_2}{\sqrt{r_1r_2}}$$

Based on 4) $\delta = \frac{2\pi}{\lambda_{cav}}(2n_{cav}L_{cav}\cos\theta_i) = 2\pi p$, $\cos\theta_i = 1$

(L_{cav} is large enough that $\phi(E)$ is neglected) (normal incidence)

$$\Rightarrow |\Delta\delta| = \frac{4\pi n_{cav}L_{cav}}{\lambda_{cav}^2}\Delta\lambda_{cav} = \frac{4\pi n_{cav}L_{cav}}{\lambda_{cav}} \cdot \frac{\Delta\lambda_{cav}}{\lambda_{cav}} = \frac{2\pi p}{Q}$$

where $Q = \frac{\lambda_{cav}}{\Delta\lambda_{cav}}$ is the quality factor of the Fabry-Perot cavity, also known as the resolving power.

At the limit of resolution, we fulfill the equality: $2\varepsilon = |\Delta\delta|$

By definition, $\gamma_c = \Delta\omega = \Delta\left(\frac{2\pi c}{\lambda}\right) = 2\pi c \frac{\Delta\lambda_{cav}}{\lambda_{cav}^2}$

$$= 2\pi c \frac{|\Delta\delta|}{4\pi n_{cav}L_{cav}} = \frac{c\cdot\varepsilon}{n_{cav}L_{cav}}$$

$$\Rightarrow \gamma_c = \frac{c(1-r_1r_2)}{n_{cav}L_{cav}\sqrt{r_1r_2}}$$

6) Adjacent modes being separated by 2π $\Rightarrow$ $F = \frac{2\pi}{2\varepsilon}$

$$\Rightarrow F = \frac{\pi\sqrt{r_1r_2}}{1-r_1r_2}$$

7) • $\vec{k} = \vec{k}_{//} + \vec{k}_z$ and the cavity photon energy is such that:

$$\vec{k}_{//}^{\,2} + \vec{k}_z^{\,2} = \left(\frac{2\pi n_{cav}}{\lambda}\right)^2$$

• under normal incidence: $\theta = \theta_i = 0$, which implies that:

$$k_z = \frac{p\pi}{L_{cav}}$$

• and the dispersion of optical modes is then given by:

$$E_c(\vec{k}_{//}) = \frac{\hbar c p\pi}{n_{cav}L_{cav}}\left(1 + \left(\frac{L_{cav}k_{//}}{p\pi}\right)^2\right)^{1/2}$$

8) For small $\vec{k}_{//}$ values: $E_c(\vec{k}_{//}) \approx \frac{\hbar c p \pi}{n_{cav} L_{cav}} \left(1 + \frac{L_{cav}^2 k_{//}^2}{2 p^2 \pi^2}\right)$

Thus, $E_c(\vec{k}_{//}) = \frac{\hbar c p \pi}{n_{cav} L_{cav}} + \frac{\hbar c \cancel{p \pi} L_{cav}^{\cancel{2}} k_{//}^2}{n_{cav} \cancel{L_{cav}} \cdot 2 p^{\cancel{2}} \pi^{\cancel{2}}}$

$$\Rightarrow E_c(\vec{k}_{//}) = E_c(0) + \frac{\hbar c L_{cav}}{2 n_{cav} p \pi} \vec{k}_{//}^2 \quad \text{with} \quad E_c(0) = \frac{\hbar c p \pi}{n_{cav} L_{cav}}$$

and finally: $E_c(\vec{k}_{//}) = E(0) + \frac{\hbar^2 \vec{k}_{//}^2}{2 m_{ph}}$ with $m_{ph} = \frac{\hbar p \pi n_{cav}}{c L_{cav}}$

9) If one considers the penetration length L_{DBR} and $\lambda_{DBR} \approx \lambda_{cav} \approx \lambda$:

$$L_{cav} + 2 L_{DBR} \approx \frac{p\lambda}{2 n_{cav}} + 2 \cdot \frac{\lambda}{2} \frac{n_2}{2 \tilde{n} \Delta n}$$

$$\approx \frac{p\lambda}{2 n_{cav}} + \frac{\lambda}{2 n_{cav}} \frac{n_{cav} n_2}{\Delta n \tilde{n}} \sim \frac{n_{cav}}{\Delta n}$$

$$\Rightarrow 2 n_{cav} (L_{cav} + 2 L_{DBR}) \approx \underbrace{\left(p + \frac{n_{cav}}{\Delta n}\right)}_{p_{eff}} \lambda$$

p_{eff} ... effective order of the cavity

$$\Rightarrow \gamma_c = \frac{c}{n_{cav}(L_{cav} + 2 L_{DBR})} \frac{1 - r_1 r_2}{\sqrt{r_1 r_2}}$$

10) a) SiO_2/Si_3N_4

The SiO_2/Si_3N_4 couple is made of amorphous materials characterized by a wide bandgap (insulating materials). They are therefore well adapted to cover a wide spectral range (from the IR down to the UV ($\lambda \sim 300$ nm)). In addition, costs associated with their deposition are low compared with crystalline semiconductor epitaxy techniques (use of low temperature ($T < 300°C$)

plasma enhanced chemical vapor deposition))

However, such a system cannot be used as a bottom DBR if we wish to realize a crystalline semiconductor microcavity (MC) unless wafer bonding techniques are employed to couple such a DBR to the cavity but it will involve lengthy processing steps to remove the DBR substrate.

(need for clean room facilities)

b) AlAs / GaAs

The AlAs/GaAs couple is a nearly strain-free system as the two materials possess relatively similar lattice parameters so that crack-free DBRs with a high number of pairs and a peak reflectivity ~100% can be obtained (while having a reduced dislocation density).

In addition, it is possible to oxidize the AlAs layers and therefore to have an oxide, AlO_x, which exhibits an optical refractive index much smaller than AlAs. As a result, AlAs/GaAs DBRs are characterized by a large stopband and can reach high reflectivities with a reduced number of pairs.

Such a couple is used to fabricate GaAs-based VCSELs emitting in the near IR. Due to the bandgap of GaAs, it is not adapted for $\lambda < 830$ nm.

c) AlN / GaN

The AlN/GaN couple exhibits a fairly large refractive index contrast so that a priori a few number of pairs is required to get high reflectivity DBRs.

However, contrary to its As-counterpart, the present couple is characterized by a significant lattice parameter mismatch which can lead to cracked structures and a high density of dislocations.

Usually the AlGaN/GaN system is preferred but it requires a much larger number of pairs due to the reduced optical refractive index contrast.

Alternatively, the AlInN/GaN system which is perfectly lattice matched for an indium content $x_{In} \sim 18\%$ can also be used, but the AlInN alloy is difficult to grow.